SIGNAL COIL CALIBRATION OF ELECTRO-MAGNETIC SEISMOMETERS

By

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ABSTRACT

We show that electro-magnetic (e-m) seismometers may be easily and accurately calibrated by removing a step of current from their signal coil, and simultaneously switching the signal coil to a recorder to capture the response. A theory is developed which obtains the damped generator constant, resonant frequency, and damping ratio from the output of a system identifier used to analyze the response. Only the seismometer mass (from the manufacturer) and the applied current (measured) need be known for a complete calibration. The coil and damping resistances are not required. The method is confirmed by comparing this signal coil method with weight lift and calibration coil calibrations. For a GS-13 V seismometer, these results were within 1.3% of each other. The undamped generator constant computed from the damped generator constant obtained by the signal coil method matched the generator constant given by the manufacturer to better than 1%. Calibration of nine new L-4C components resulted in undamped generator constants all within 3% of the values given by the manufacturer. The circuit used in the signal coil method is shown and explained.

INTRODUCTION

Many electromagnetic (e-m) seismometers and almost all geophones lack calibration coils, and so must be calibrated by other means. Classic techniques for calibrating such transducers include weight-lift and shake-table methods, and the more complicated Willmore bridge method (Willmore, 1959; Barr, 1964; Donato, 1971). Seismometers may also be calibrated by comparison with a colocated, previously calibrated seismometer (Vernon and Pavlis, 1994), or by measuring the phase between a sinusoidal voltage applied to the signal coil and the resulting current (Liu and Peselnick, 1986). The method we favor is the step-release-method which measures the transient response of the signal coil when a step of current is released (Asten, 1977; Houliston *et. al*., 1982; MacArthur, 1985; Menke *et. al*., 1991). The resulting transient has a simple analytic dependence on the damped generator constant, resonant frequency, and damping of the seismometer and can be analyzed to determine the values of these parameters. Asten, *et. al*. (1977) obtain the seismometer parameters directly from properties of the step-release transient, and Menke *et. al*. (1991) from its spectrum. Despite the availability of these calibration methods, because of their complexity or difficulty many users simply use the specifications supplied by the manufacturer, which is a less than satisfactory practice.

This work demonstrates that the step-release method referred to above, when used with a system identifier, is a simple, fast, and very accurate method for calibrating seismometers and geophones .

CALIBRATION DEFINED

A seismometer is calibrated when the three parameters which define the velocity sensitivity are known. The velocity sensitivity, VS, is given by the ratio of the Laplace transform of the output voltage at the seismometer terminals, $E(s)_{out}$, to the transform of the input ground velocity, $\dot{X}(s)$, (Riedesel *et. al.*, 1990; Aki and Richards, 1980). All initial conditions are assumed to be zero. The VS is given by equation (1) below. The three defining parameters are the damped generator constant, G_d, the resonant frequency, $f_0 = \frac{\Omega}{2\pi}$, and the fractional damping ratio, ζ.

$$VS(s) = \frac{E(s)_{out}}{\dot{X}(s)} = \frac{G_d s^2}{s^2 + 2\zeta\Omega s + \Omega^2} \qquad \frac{V}{m/s} \qquad (1)$$

The damped generator constant, G_d, is related to the open circuit or undamped generator constant of the signal coil, G_{sig}, by:

$$G_d = \frac{r_d}{r_c + r_d} G_{sig} \qquad \frac{V}{m/s} \qquad (2)$$

where r_c and r_d are the resistances of the signal coil and damping resistor, respectively.

This form for the VS assumes that the coil inductance is small enough that there is no low-pass corner frequency near the frequency band of interest, and that the low-pass corner of any recorder or data-logger occurs at a frequency very much larger than f_0. It also assumes that the input impedance of the recorder is very much greater than the parallel combination of r_c and r_d. To achieve this in some situations may require placing a high input impedance buffer amplifier between the seismometer and the recorder. The amplitude spectrum, $|VS(f)|$, is shown in Figure 3(a), which indicates the level of the high frequency gain, G_d, and the location of the resonant frequency, f_0.

CALIBRATION USING A STEP OF CURRENT

We simplify the analysis of the current release response of the seismometer by considering instead the response to a step of applied current. By considering the applied step case, we are able to work with zero initial conditions. In practice, when calibrating using a calibration coil, either a step application or release may be used because the small back emf generated by the tiny motor constant of the calibration coil does not diminish the applied current. However, when the current is applied to the signal coil with its (possibly) large generator constant, the back emf can be large and thereby reduce the applied current significantly. The effects of this back emf on the accuracy of calibration has been treated by Rodgers (1992). In addition, the applied dc current produces a voltage drop which is applied directly to the data recorder corrupting the recording of the response. For both these reasons a current release must be used when a current is applied directly to the signal coil.

(a) Current Applied to Calibration Coil

In developing the seismometer response to a step of current step applied to its signal coil, it is convenient to first examine the response to a step of current applied to its calibration coil. The transform of the response, $E(s)_{step.cal}$, to a step of current, I_{cal}, applied to the calibration coil having a motor constant G_{cal}, is given by equation (3), (Berg and Chesley, 1976) :

$$E(s)_{step.cal} = \frac{K_1}{s^2 + 2\zeta\Omega s + \Omega^2} \qquad \text{V / Hz} \qquad (3)$$

where:

$$K_1 = \frac{G_d G_{cal}}{M} I_{cal} \qquad (4)$$

(b) Current Applied to Signal Coil

When a step of current, I_{sig}, is applied directly to the signal coil of a seismometer, the mass is displaced from its null position by a step of force in Newtons equal to $G_{sig}I_{sig}$. The reason for this is that in the mks system the generator constant of a coil-magnet pair in V / m / s is exactly equal to its motor constant in Newtons / A. This may be confirmed by equating electrical power in watts , V*A, to mechanical power in Joules / sec which are Newton-m / sec. So the response of the seismometer to a step of current applied to its signal coil is proportional to the response to a step of current applied to its calibration coil except that the motor constant, G_{cal} is replaced by the undamped generator constant, G_{sig} . This is shown in equations (5) and (6), where $E(s)_{step.sig}$ is the transform of the step response. K in equation (6) is analogous to K_1 in equation (4) with G_{cal} replaced by G_{sig} and I_{cal} by I_{sig}.

$$E(s)_{step.sig} = \frac{K}{s^2 + 2\zeta\Omega s + \Omega^2} \tag{5}$$

$$K = \frac{G_d G_{sig}}{M} I_{sig} \tag{6}$$

The actual temporal response of the seismometer to the applied step of current is given by the inverse transform of equation (5), $e(t)_{step.sig}$:

$$e(t)_{step.sig} = \frac{K}{\Omega\sqrt{1-\zeta^2}} e^{-\zeta\Omega t} \sin\left(\Omega\sqrt{1-\zeta^2}\, t\right) u(t) \tag{7}$$

where $u(t)$ is the unit step function and $0 < \zeta < 1$. The analytic expressions for $e(t)_{step.sig}$ for $\zeta \geq 1$ are not shown. An example of the form of $e(t)_{step.sig}$ is shown in Figure 2 for an underdamped Teledyne GS-13 seismometer. Notice that the pre-response baseline is at zero volts, which would not be the case if the recorder were connected while the current step was on. The method used to record these data is described in the next section.

RECORDING CIRCUIT

As mentioned previously, the proper recording of the response to the removal of a step of current applied directly to the signal coil requires that the signal be zero prior to the removal of the current. But because I_{sig} is flowing through the signal coil, a dc voltage drop $I_{sig}\, r_c$ appears at the seismometer terminals. This must be removed prior to the termination of the current. This is accomplished by the switching circuit shown in Figure 1. Initially, the switch

SW1 is to the left so that an external dc current, I_{ext}, is applied to the signal coil terminals of the seismometer. This current divides between the damping and signal coil resistors resulting in a dc current, I_{sig}, flowing through the signal coil displacing the mass as described earlier. I_{sig} terminates when SW1 is switched to the right. This action simultaneously connects the seismometer terminals to the recorder enabling it to record the step-release transient from the signal coil of the seismometer. An example of data recorded by this method is the barely visible dash-dot trace shown in Figure 2. In practice, in order to reduce ambient noise pickup, SW1 in Figure 1 is a double-pole-double-throw (DPDT) switch in which the other half is used to ground the recorder input until the moment of switching.

FINDING G_d, f_0, AND ζ FROM $e(t)_{step.sig}$

When a seismometer can be characterized by a small number of parameters, such as in this case, these parameters can be accurately estimated by a system identification program (*cf.* Chapman, *et. al.* , 1988). The particular system identifier which we use is a time-domain seismometer identifier called ID (Harris, 1994). ID is available on the Internet via anonymous ftp from:

quake.crustal.ucsb.edu:/scec/sun/llnlid-83.334.exe.tar.Z

ID produces estimates of K, f_0 and ζ , which we call $\hat{K}$, $\hat{f}_0$, and $\hat{\zeta}$, by optimizing the parameter values to minimize the mean-squared error between the recorded data and the time-domain model of equation (7). The resonant frequency and damping are determined directly from $f_0 = \hat{f}_0$, and $\zeta = \hat{\zeta}$. However, following equation (6), the damped generator constant must be obtained from:

$$\hat{K} = \frac{G_d G_{sig}}{M} I_{sig} \qquad (8)$$

In equation (8), G_{sig} can be eliminated since G_{sig} and G_d are related by:

$$G_{sig} = \frac{r_c + r_d}{r_d} G_d \tag{9}$$

Also, referring to the circuit diagram of Figure 1, I_{sig} and I_{ext} are related by:

$$I_{sig} = \frac{r_d}{r_c + r_d} I_{ext} \tag{10}$$

This relation may be used to eliminate I_{sig} . Substituting equations (9) and (10) into equation (8), and solving for G_d results in the simple expression :

$$G_d = \left(\frac{M}{I_{ext}} \hat{K} \right)^{1/2} \tag{11}$$

We draw several conclusions from equation (11) about the advantages of the signal coil method of calibration. The method requires fewer manufacturer-supplied constants than calibration coil methods, and is correspondingly more accurate. Only the seismometer mass, M, need be known from the manufacturer; the remaining parameters are either measured (I_{ext}) or estimated by the system identifier ($\hat{K}$). The calibration coil method requires the manufacturer-supplied calibration coil motor constant, G_{cal}, , which may be inaccurate. Another significant advantage of the signal coil method is that it requires no knowledge of the signal and damping coil resistances, r_c and r_d. This fact is especially useful when the coil resistance is unknown and the seismometer, including the damping resistor, is inaccessible as in a borehole or is other wise inaccessible.

Equation (11) applies to seismometers having rectilinear suspensions. For pendulous seismometers, the expression for G_d becomes:

$$G_d = \left(\frac{d_m}{d_{sig}} \frac{M}{I_{ext}} \hat{K} \right)^{1/2} \qquad (12)$$

where d_m and d_{sig} are the distances from the hinge to the center of mass of the pendulum and to the signal coil, respectively.

One way of testing the signal coil method for accuracy is to compute the undamped signal coil generator constant , G_{sig}, and compare this with the generator constant, G_{mfg}, supplied by the manufacturer. G_{sig} is obtained from equations (9) and (11):

$$G_{sig} = \frac{r_c + r_d}{r_d} \left(\frac{M}{I_{ext}} \hat{K} \right)^{1/2} \qquad (13)$$

These comparisons will be made later for several seismometers.

SETTING THE LEVEL OF I_{ext}

In order to minimize the degradation in Signal-to- Noise Ratio (SNR) due to ambient seismic background noise, I_{ext} should be made large enough so that the mass is displaced a considerable fraction of its maximum linear travel. To find the value of I_{ext} required to produce a mass displacement z, solve for the spring stiffness, k, from the relation:

$$\Omega^2 = (2\pi f_0)^2 = \frac{k}{M} \qquad (14)$$

The force, F, on the mass is $F = G_{sig} I_{sig}$. This is balanced by the force $F = kz$. So the resulting displacement, z, is found from:

$$z = \frac{F}{k} = \frac{G_{sig} I_{sig}}{(2\pi f_0)^2 M} \qquad (15)$$

Substituting for I_{sig} from equation (10), and solving for I_{ext} results in:

$$I_{ext} = 4\pi^2 \frac{r_c + r_d}{r_d} M \frac{f_0^2}{G_{sig}} z \qquad (16)$$

A reasonable value to use for z might be one-half of the maximum zero to peak linear travel. For seismometers having large generator constants, such as the Teledyne GS-13, this will result in the generation of large voltages so that attenuation must be used to avoid saturation of the recorder. One way to include attenuation is to substitute a resistor equal to the damping resistor but which has a low voltage tap on it that is connected to the recorder. Whatever method is used to incorporate attenuation, it is important not to load the signal coil and damping resistances. In any case, the attenuation term would appear dividing the term $\hat{K}$ in equations (12) and (13).

RESULTS

(a) Verification of the Signal Coil Calibration Method Using a Teledyne GS-13 V

Using a circuit similar to that shown in Figure 1, three calibration experiments were run on a Teledyne GS-13 vertical seismometer. In order to verify that the signal coil calibration method is valid, and that the expression for G_d given by equation (11) is indeed correct , three types of calibrations were performed: a weight lift, releasing a current step from the calibration coil, and releasing a current step from the signal coil. The transient response resulting from releasing a current step from the signal coil is shown as the barely visible dash-dot line in Figure 2. The solid line in Figure 2 is the model which ID fit to the data. The fit is such that the two curves are nearly indistinguishable. No ambient noise is visible prior to the response because the input to the recorder is shorted, and none is visible following the response because a large mass displacement was used resulting in a large signal voltage. Using equation (9), a G_{sig} is computed from each of the three calibrations. These are compared with the value of G_{sig} supplied by the manufacturer, which is designated G_{mfg} and is taken to be the reference value. The results are given in Table 1 below which shows that the G_{sig} computed by the signal coil calibration method was within 0.99% of the manufacturers value, G_{mfg} = 2353 V / m / s. The weight lift and calibration coil methods produced G_{sig} values which were within 1.3 % and 0.70% of G_{mfg} , respectively. Variations between the measured resonant frequencies and damping ratios from the three calibrations were all less than 1.8%. The results shown in Table 1 are taken as confirmation of the theory and accuracy of the signal coil method. Finally, using the results from the signal coil calibration method, the amplitude spectrum of the velocity sensitivity, $|VS(f)|$, is plotted in Figure 3(a). The associated phase advance and group delay curves are shown as Figures 3(b), and 3(c), respectively. The phase shift, $\Theta(\omega)$, is given by:

$$\Theta(\omega) = \tan^{-1}\left(\frac{\operatorname{Im} VS(\omega)}{\operatorname{Re} VS(\omega)}\right) = \pi - \tan^{-1}\left(\frac{2\zeta\Omega\omega}{\Omega^2 - \omega^2}\right) \qquad (17)$$

The phase advance was obtained from the phase shift by dividing the phase shift by $2\pi f$. . The term Phase Advance is used because VS(s) is high-pass and does indeed advance a steady state sinusoid in time, not delay it.

The group delay, T_{gr}, was obtained by differentiating the phase shift with respect to ω:

$$T_{gr} = \left(\frac{1}{\pi}\right) \cdot \frac{f_0 \zeta\, f^2 + f_0^3 \zeta}{f^4 + 2\, f_0^2\, f^2 (2\zeta^2 - 1) + f_0^4} \qquad (18)$$

Expressions (17) and (18) are plotted in Fig. 3(b) and 3(c), respectively.

(b) Signal Coil Calibration Method Applied to Three Mark Products L-4C-3D Seismometers

In order to further verify the signal coil method, it was used to calibrate three new Mark Products L-4C-3D seismometers. The results are shown in Table 2 below. As in Table 1, the percent errors in G_{sig} were computed using G_{mfg} as the reference value. All the errors were found to be less than 2.9%, which is slightly higher than the result for the GS-13, but still within a very acceptable range. The values for the resonant frequencies, f_0, and the damping ratio, ζ, are also given and appear to be reasonable.

CONCLUSIONS

We have shown that by combining the infrequently used technique of removing a current step from the signal coil, switching the signal coil, and analyzing the response with a robust system identifier can result in simple to perform and accurate (1% - 3%) calibrations of e-m seismometers. It has the additional and unexpected advantage that the only data required from the manufacturer is the mass. Even the coil and damping resistances are not required for a complete calibration.

The limitations on the signal coil method are imposed by the system identifier which requires that any low pass corner frequency be much larger than the seismometer resonant frequency; however, this is a soft constraint which can be removed by altering the system identifier.

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TABLE 1

Comparison of calibrations using weight lift, calibration coil, and signal coil methods.

Seismometer GS-13 V , SN # 290				
	G_{sig} Weight Lift	G_{sig} Calibration Coil	G_{sig} Signal Coil	G_{mfg}
V / m /s	2299.4	2369.5	2329.7	2353
% error	1.31%	0.70%	-0.99%	
f_0 , Hz	1.08	1.10	1.09	
ζ	0.68	0.67	0.66	

TABLE 2

Signal Coil Calibration Method Applied to Three L-4C-3D Seismometers: SN's 1199, 1200, & 1201

SN Axis	# 1199 V	# 1199 L	# 1199 T	# 1200 V	#1200 L	#1200 T	#1201 V	#1201 L	#1201 T
G_{sig} (meas.)	712.8	697.4	697.1	697.6	695.7	686.6	704.9	684.2	704.3
G_{mfg}	696.8	692.9	696.8	681.1	696.8	689.0	700.8	704.7	700.8
% Error	2.3%	0.65%	0.04%	2.42%	-0.16%	-0.35%	0.58%	-2.91%	0.49%
f_0 , Hz	1.03	1.03	1.03	1.08	1.11	0.99	1.19	1.07	1.08
ζ	0.81	0.76	0.76	0.75	0.732	0.78	0.71	0.76	0.76

CAPTIONS

Figure 1.

This is the recording circuit used to capture the response of the seismometer to the release of a step of current of amplitude I_{ext} . Initially, switch SW1 is to the left allowing I_{ext} to flow into the seismometer resulting in a current I_{sig} through r_c. The mass is displaced by a force of $G_{sig}\, I_{sig}$ Newtons. The mass is released and the response simultaneously recorded when SW1 is switched to the right. In practice, SW1 is a double-pole-double throw (DPDT) switch one-half of which is used to ground the input to the recorder until the switch is thrown.

Figure 2.

The response of a GS-13 V seismometer to the release of a step of current of 220 μA applied to its signal coil is shown by the barely visible dash-dot line. The solid line is the model which the system identifier ID fit to the data. No ambient noise is visible prior to the response because the input to the recorder is shorted, and none is visible following the response because a large mass displacement was used resulting in a large signal voltage. These data are the basis for the computed seismometer parameters given in Table 1.

Figures 3(a), 3(b), and 3(c)

The amplitude spectrum, $|VS(f)|$, of the velocity sensitivity of the GS-13 V, SN #290, seismometer is shown as Figure 3(a). This was obtained from equation (1) using the parameters obtained from running ID on the step response shown as the dot-dash line in Figure 2. The applied external current, I_{ext} = 220 μA. The parameters obtained were G_d = 2152.4 V / m / s, f_0 = 1.09 Hz, ζ = 0.66. The associated Phase Advance and Group Delay are shown as Figures 3(b), and 3(c), respectively.